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THESIS

TIMETABLING COURSES AT THE
NAVAL POSTGRADUATE SCHOOL

by

Francisco J. Hederra

September, 1994

Thesis Advisor: Robert F. Dell

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The Naval Postgraduate School (NPS) course schedulers use a time-consuming manual process to assign courses, students, and professors to classrooms. The 1994 NPS Winter Quarter had approximately 535 courses, 953 student-groups, and 230 faculty members assigned to approximately 100 classrooms. This thesis formulates the NPS course timetabling problem as a mixed integer linear problem and develops a Lagrangean relaxation based heuristic to assist the schedulers. The heuristic requires approximately 15 IBM/RISC/6000 model 590 CPU hours to obtain a timetable for the 1994 Winter Quarter (compared to six weeks for the equivalent manual exercise). Results indicate that the heuristic can be used successfully to support the study of policy questions. Studies conducted in this thesis show the effect of decreasing classrooms and both increasing and decreasing the number of students.

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Timetabling Courses at the
Naval Postgraduate School

by

Francisco J. Hederra
Lieutenant, Chilean Navy
B.S. Electrical Engineering, Escuela de Operaciones, 1989

Submitted in partial fulfillment
of the requirements for the degree of

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Author: ______________________
Francisco J. Hederra

Approved by: ______________________
Robert F. Dell, Thesis Advisor

____
Gordon Bradley, Second Reader

____
Peter Purdue, Chairman
Department of Operations Research

ABSTRACT

The Naval Postgraduate School (NPS) course schedulers use a time-consuming manual process to assign courses, students, and professors to classrooms. The 1994 NPS Winter Quarter had approximately 535 courses, 953 student-groups, and 230 faculty members assigned to approximately 100 classrooms. This thesis formulates the NPS course timetabling problem as a mixed integer linear problem and develops a Lagrangean relaxation based heuristic to assist the schedulers. The heuristic requires approximately 15 IBM/RISC/6000 model 590 CPU hours to obtain a timetable for the 1994 Winter Quarter (compared to six weeks for the equivalent manual exercise). Results indicate that the heuristic can be used successfully to support the study of policy questions. Studies conducted in this thesis show the effect of decreasing classrooms and both increasing and decreasing the number of students.

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EXECUTIVE SUMMARY

The Naval Postgraduate School (NPS) course schedulers use a time-consuming process to assign courses, students and professors to classrooms. Unlike its civilian counterparts, NPS attempts to accommodate every student's quarterly requests for courses. This policy creates a complicated timetabling problem with approximately 1,000 student groups each taking a different subset of approximately 500 courses. The NPS course schedulers must assign the student groups and courses without conflict into approximately 100 classrooms available for nine daily periods, five days a week.

In addition to potential student conflicts, the timetabling process is complicated by a number of other preferences which the course schedulers try to accommodate. These include:

- Requests for specific classrooms, periods, or days,
- Lunchtime for students and faculty between 1100 and 1400,
- Professor's desires for back to back teaching or breaks between teaching courses (the typical professor teaches two courses, two quarters a year),
- Proximity of the assigned classroom to the instructor's office,
- Courses requiring blocks of two, or three consecutive hours,
- etc.

These potential conflicting preferences reduce the freedom to timetable and often result in the course schedulers being unable to identify a solution which fully satisfies all of them.

Starting with the data used to timetable the 1994 Winter Quarter at NPS, this thesis explores a reduction in the number of variables needed to describe the problem using grouping techniques for students and for classrooms. Additional reductions are obtained by defining a set of patterns on which courses can be timetabled during the week according to their required number of hours. The problem is then formulated as a Mixed Integer Problem, MIP, which is run on an AMHDAL 5995 using commercial optimization software. After 48 AMHDAL 5995 CPU hours, the software was unable to solve the linear programming relaxation. This result provided motivation for our heuristic development. A Lagrangean relaxation based heuristic is formulated and implemented to solve the model on an IBM/RISC/6000 model 590. The heuristic requires approximately 15 CPU hours to obtain a timetable for the 1994 Winter Quarter (compared to six weeks for the equivalent manual exercise). Results indicate that the heuristic can be used successfully to support the study of policy questions. Studies conducted in this thesis show the effect of decreasing classrooms and both increasing and decreasing the number of students.

I. INTRODUCTION

The Naval Postgraduate School (NPS) course schedulers currently use a time-consuming manual process to assign courses, students and professors to classrooms. Unlike its civilian counterparts, NPS attempts to accommodate every student's quarterly request for courses. This policy creates a complicated quarterly timetabling problem with approximately 1,000 student groups, each taking a different subset of approximately 500 courses. The NPS course schedulers assign these courses into approximately 100 classrooms without student or room conflict while observing side constraints and satisfying faculty preferences. This thesis develops a Lagrangean relaxation based heuristic to assist the schedulers timetable courses. The heuristic's primary intent is to help the schedulers perform policy studies which look at the effect of classroom reductions and variations in the number of students.

A. THE NAVAL POSTGRADUATE SCHOOL

1. Mission and Organization

The NPS mission is to provide advanced professional studies at the graduate level for military officers and defense officials from all United States (US) services and other nations. In 1994 the student population was

approximately 1580 US and 236 International students coming from 40 different countries. Of these there were 16 civilians and 1797 officers.

At NPS there are two organizations (Curricular Offices and Academic Departments or Groups) that support student instruction. The NPS has eleven Academic Departments and four interdisciplinary groups that operate like departments at most civilian universities. The Curricular Offices are organizational entities separated from, but interactive with, the Academic Departments. The Curricular Offices primarily perform three functions:

- Academic counseling and military supervision of Officer students;
- Curriculum development and management to ensure attainment of professional and academic objectives; and
- Liaison with curricular sponsor representatives (every curriculum has a Department of Defense Agency that ensures that the courses taught and their objectives agree with service needs).

2. NPS Academic Year

The NPS operates year-round offering courses in four quarters (Fall, Winter, Spring, Summer) consisting of eleven weeks of instruction and one week for final examinations. Every academic week consist of five days (Monday through Friday) each with nine 50 minute-long teaching periods (0810 - 1700). Every quarter during this academic year, a certain

number of courses are offered which may or may not be the same as those offered in any previous quarter.

B. COURSE TIMETABLING AT NPS

1. Current Timetabling Process

Quarterly courses at NPS are categorized as regular, refresher, accelerated, or seminars, the first ones comprise the bulk of the courses and last 12 consecutive weeks. Refresher courses are undergraduate level and are taught either for the entire quarter or for the last six weeks. Accelerated courses are offered only for the first six weeks of the quarter. Finally, seminars are meetings, lectures, etc., that students and/or faculty attend on predetermined days in predetermined classrooms and periods.

For all regular, accelerated and refresher courses, NPS has a policy of having no more than 30 students per course segment, consequently some high demand courses must be divided into more than one segment. For timetabling purposes, regardless of its type any course segment is treated separately.

Prior to the beginning of any quarter, students pick approved courses they desire to take. The later assignment of students to course segments is done usually by the registrar's office during the timetabling process (Nolan,1992 p.65) while faculty members are assigned to courses by the Academic Departments prior to that process.

The timetabling process at NPS consists basically of four phases (Nolan, 1992 p.6): Forecasting, PreScheduling, Scheduling, and PostScheduling. The first two phases are oriented to produce the input data for the third phase by determining the courses, course segments and their instructors based on the NPS Catalog, student requirements, and instructor availability. The third phase consists of course and final examination timetabling. During this phase, the schedulers attempt to produce a timetable without students conflicts while observing every course requirement and faculty preference (constraints). If this objective is achieved, the schedulers proceed to make a feasible classroom assignment and thereby end the process. If the objective is not reached the schedulers try to lift the hardest constraints (such as cancelling offered courses, changing required periods, etc.) using direct coordination with the Curricular Offices and Academic Departments (Nolan, 1992 p.89) and then they attempt again to get a feasible timetable in an iterative process that ends with the Master Schedule. This Master Schedule may suffer some minor changes of classrooms and time periods that are managed by the schedulers on what is called the PostScheduling phase.

At NPS there is a Registrar's Office where two of its members currently spend six weeks, every quarter, planning the next period's timetable. Their job is completely manual.

Equipped with paper, pencils, and a lot of erasers, they rely on previous experience to obtain the Master Schedule.

2. Constraints

The NPS timetabling problem's constraints are of two types:

a) Required constraints :

- The Master Schedule must be feasible for all students, *i.e.* every student must be able to attend all the courses s/he signed for.
- The Master Schedule must be feasible for all faculty, *i.e.* every faculty member must be able to teach his/her assigned courses.
- There must be at least one hour reserved for each faculty member and student between 1100 and 1400 for lunch.
- A course may require a particular classroom, *e.g.* lab.
- A faculty member may require a particular period or day to teach his course.
- Every course must be timetabled for the weekly number of hours specified in the catalog.
- The assigned classroom must have sufficient seat capacity for the attending number of students.
- Courses are timetabled only one session per day (maybe for more than one period).
- Departments may block out time for seminars or faculty meetings.

b) Preference constraints: These are constraints which may be violated.

- Some faculty members prefer teaching courses back to back or with breaks between their courses.
- Most faculty members prefer teaching in rooms close to their office.

- Some faculty members prefer teaching in blocks of two or three consecutive hours.
- Some faculty members prefer teaching the same period in the same classroom every day of the week.

3. Dimensions of the Problem

During the 1994 Winter Quarter, the NPS schedulers manually solved a problem with the following dimensions:

- 104 classrooms;
- 953 student groups;
- 535 offered courses (all courses that require a classroom assignment included) where 177 of them required a lab period. At the end of the NPS timetabling process, 30 courses were not timetabled most of them due to lack of students;
- 230 faculty members were assigned to teach the above courses;
- Every student signed for a minimum of one course and a maximum of five courses making a total of 4483 student-group-course combinations.

Considering that a course could be timetabled in a classroom during any of the nine daily periods and in any of the five week days, an unconstrained timetabling model would have roughly 2,503,800 variables (535 x 104 x 9 x 5). The number of constraints is also large (a model implemented in this thesis has 42,885 constraints only to prevent student conflicts).

C. THESIS OUTLINE

This thesis discusses both a mathematical model and empirical results obtained for the NPS quarterly course timetabling problem using a Lagrangean relaxation based heuristic. Chapter II of this thesis covers a brief description of similar timetabling problems found in the operations research literature and the different techniques that have been employed to solve them. In Chapter III the NPS timetabling problem is introduced, including a description of the grouping strategies adopted to reduced the number of variables. This chapter also discusses a Lagrangean relaxation based heuristic and a method to solve the Lagrangean relaxation subproblem. In Chapter IV the results of these different techniques are explained. Chapter V contains conclusions. The Appendix provides a listing of classroom groups.

II. RELATED LITERATURE

Course timetabling problems have been widely analyzed in the operations research literature since the late 1950's (Tripathy, 1984 p. 1473). These problems arise naturally in different academic organizations in the form of courses, final exams, or seminar subjects to be timetabled.

The basic problem consists of determining an optimal assignment of courses to classrooms for certain periods while satisfying side constraints. The optimality condition is in some cases satisfied just with feasibility whereas in some other cases, preferences for periods are added to the objective function.

In any of its versions, the course timetabling problem displays the following characteristics:

- The objective function contains either desires (cost) to timetable a course on a given period or the number of conflicts produced by a given timetable. In the first case the problem direction is to maximize whereas in the second case it is to minimize.
- It has a fixed number of courses to be offered with some predetermined number of hours to be timetabled.
- It has a fixed number of students that choose a set of courses among the list of offered courses. In only some cases the solution attempts to fully satisfy the student requests.
- A fixed number of periods are available to timetable the courses. For some cases these periods are not of the same

duration and for others the courses do not necessarily start at the same time.

- A fixed number of rooms are available to timetable the courses and each room has a fixed student capacity.

The NPS quarterly timetabling problem has a size that could be compared to that of other universities but differs with most models reported in the literature since it attempts to satisfy all students' course requests. The NPS problem was analyzed by Nolan and Youngblood (1992) who suggest a simulated annealing based heuristic as developed by D. Erickson. This Heuristic reportedly reduced student conflicts by 90%. Unfortunately, details about Erickson's simulated annealing implementation and computational testing are not available. Another report, Fiegas (1985), suggests a heuristic based on solving iterative integer linear programs for the NPS scheduler. Fiegas was not able to implement or test this procedure. The NPS final exam timetabling was solved by Golmayo (1994).

Literature related to civilian timetabling includes university timetabling by Tripathy (1984) using Lagrangean relaxation combined with a Branch and Bound method to solve a problem of 94 courses, 33 student groups and five rooms over 18 and 36 periods per week. Fahrion and Dollansky (1992) approached the problem using PROLOG, an artificial intelligence programming language, to solve a problem with 119 courses, 21 rooms and 43 instructors using a personal

computer. Dowsland (1990) uses a graph coloring heuristic to determine the minimum number of weekly time-slots necessary for a full timetable and to find the timetable that minimizes a measure of disappointment for the case of 100 students and some 30 courses. Seminars and congress topics are covered by Barham and Westwood (1978) for a problem of 22 optional courses offered to 36 students. Their objective was to determine the minimum number of periods required to satisfy a primary list of selections for every student. To solve it they used heuristic assignment based on the number of conflicts that a course would produce. The same problem was pursued further by Tripathy (1980) using Lagrangean relaxation, Branch and Bound procedures and an Out of Kilter algorithm. Finally, university examination timetabling was analyzed by Johnson (1990) for 200 exams to be timetabled in 20 periods with around 2350 students taking from three or four exams each using a pure heuristic approach based on weighing courses according to the number of potential course or student conflicts.

The generic problem was treated by Ferland and Lavoie (1992) using exchange procedures and by Hertz (1992) using tabu search. The difficulty of the classroom assignment problem is analyzed extensively by Carter and Tovey (1992) for three different types of objectives: feasibility, *i.e.* every course always in the same room and no more than one course per room simultaneously, satisfice, *i.e.* feasibility plus each

class in a satisfactory room, and Optimize, *i.e.* considering a cost to assign a course to a particular room. They even analyzed situations with different sizes or with included preferences. They categorized the problems on whether the objective was to timetable a course only once a week, which they named the interval problem, or more than once a week, which they defined as the noninterval problem. They concluded that the interval problem was polynomial solvable even for the optimization problem but that the noninterval classroom assignment feasibility problem, and therefore the satisfice and optimize problems, was NP complete. It is important to consider that the classroom assignment is only part of the NPS course timetabling problem which includes at least one extra set of constraints to ensure student feasibility.

III. MODELS

A. INTRODUCTION

The NPS course timetabling problem is the assignment of courses to classrooms for specific weekday periods such that all the student course requirements are satisfied in a non-conflicting timetable. This non-conflicting requirement considers time constraints for students, rooms, and faculty in such a way that the course can effectively be taught without any interference. With this in mind, the set of constraints guarantee feasibility and the objective function accounts for preferences constraints.

B. GROUPING TECHNIQUES

In order to reduce the number of variables in the problem, the following grouping techniques are exploited:

Student Groups: The NPS schedulers group students that have requested the same set of courses and belong to the same curricular section. The number of student groups impacts on the amount of work required to solve the timetabling problem, so this thesis implements a routine to reduce it. The number of groups can be reduced by combining students from different curriculums and also by adding students who have a subset of the group's set of courses. For example if students A,B,C respectively request the set of courses K_A, K_B, and K_C where K_B

$\subseteq K_A$ and $K_C \subseteq K_A$, then a feasible timetable for A is also feasible for B and C. Therefore A, B and C are grouped as a single student group. This is considered in the data set developed for the analysis of the NPS problem (1994 Winter Quarter) and provides a reduction from 953 to 725 student groups and a reduction in the student-group-course combinations from 4483 to 3767.

Classroom Grouping: A course must be timetabled in a room satisfying equipment needs and with capacity larger than the number of attendees. It is also preferred for a course to be timetabled close to the instructor's office. These constraints and preferences can be achieved by defining sets of classrooms sharing capacity, location, and equipment characteristics.

For the NPS case, classrooms are categorized according to their purpose, location and capacity. Afterwards all rooms belonging to the same category are grouped and their components counted to determine the cardinality of this set (the Appendix shows the results of this process). Correspondingly, the classroom feasibility constraints are oriented to not exceed the cardinality of the room category set.

Mode Grouping: Courses typically meet once a day during the same period. The possible combinations of days to arrange course meetings, on what this thesis refers to as modes are therefore a function of the number of sessions that a course meets. For example a three session per week course can be

timetabled in the following modes: Monday-Tuesday-Wednesday, Monday-Tuesday-Thursday, Monday-Tuesday-Friday, Tuesday-Wednesday-Thursday, Tuesday-Wednesday-Friday, Wednesday-Thursday-Friday, Monday-Wednesday-Thursday, Monday-Wednesday-Friday, Monday-Thursday-Friday, or Tuesday-Thursday-Friday.

For the NPS case, previous timetabling experience shows that the preference constraint for meeting during the same period are almost always met (37 out of 470 courses in 1994's Winter Quarter did not) and that courses are timetabled using a limited subset of all available modes.

Blocks: A block corresponds to the number of consecutive hours that a course meets daily, is a preference of the faculty member teaching the course, and is always less than four hours. Block length and catalog number of hours determines the number of times a course will meet weekly and therefore the feasible timetabling modes.

C. MODEL

The model of the NPS course timetabling problem is presented below

INDICES

a = classroom category,
m = course mode,
d = day of the week,
f = faculty member,
k = course,
p = period of the day,
s = student group.

DATA

Student related:

COU_s = Set of courses common to student group s.

Course related:

$GROUP_k$ = Set of classrooms categories where course k can be timetabled,

$MODE_k$ = Set of modes when course k can be timetabled,

PER_k = Set of periods when course k can be taught,

$PERDS_{kp}$ = Set of periods that course k could start and still be in session during period p (for example a two hour block course started in period p-1 or p would be in session in period p).

$LUNCHP_k$ = Set of periods that course k can start and still be in session during lunch periods,

$CARD_a$ = Number of classrooms in category a,

$PREF_{kapm}$ = Course k's preference for period p, mode m and classroom group a,

$MODEDAY_d$= Set of course modes that meet on day d.

Faculty member related:

FAC_f = Set of courses that faculty member f teaches,

$FACSTU_f$ = Set of courses that faculty member f attends as a student.

VARIABLES

X_{kapm} Binary variable with value one if course k is timetabled in classroom category a starting in period p using mode m.

$EL1_{apd}$ Elastic variable which indicates that additional classrooms of type a are needed for period p and day d.

$EL2_{spd}$ Elastic variable which indicates if more than one course is timetabled simultaneously for student group s on period p and day d.

$EL3_{fpd}$ Elastic variable which indicates if more than one course is timetabled simultaneously for faculty f on period p and day d.

FORMULATION

MINIMIZE

$$(0) \quad \sum_{p} \sum_{d} \left(\sum_{a} EL1_{apd} + \sum_{s} EL2_{spd} + \sum_{f} EL3_{fpd}\right) - \sum_{k} \sum_{(a \in GROUP_k)} \sum_{(m \in MODE_k)} \sum_{p} PREF_{kapm} X_{kapm}$$

SUBJECT TO:

$$(1) \quad \sum_{(a \in group_k)} \sum_{(p \in PER_k)} \sum_{(m \in MODE_k)} X_{kapm} = 1 \qquad \forall k$$

$$(2) \quad \sum_{(k \,/\, p \in PER_k) \text{ and } (k \in COU_s)} \sum_{(a \in GROUP_k)} \sum_{(m \in MODEDAY_d \text{ and } MODE_k)} \sum_{(p' \in PERDS_{kp})} X_{kap'm} \leq 1 + EL2_{spd} \qquad \forall s, p, d$$

$$(3) \quad \sum_{(k \,/\, p \in PER_k \text{ and } a \in GROUP_k)} \sum_{(m \in MODEDAY_d \text{ and } MODE_k)} \sum_{(p' \in PERDS_{kp})} X_{kap'm} \leq CARD_a + EL1_{apd} \qquad \forall a, p, d$$

$$(4) \quad \sum_{(k \,/\, p \in PER_k) \text{ and } (k \in FAC_f \cup FACSTU_f)} \sum_{(a \in GROUP_k)} \sum_{(m \in MODEDAY_d \text{ and } MODE_k)} \sum_{(p' \in PERDS_{kp})} X_{kap'm} \leq 1 + EL3_{fpd} \qquad \forall f, p, d$$

$$(5) \quad \sum_{(k \in COU_s)} \sum_{(a \in GROUP_k)} \sum_{(p \in LUNCHP_k)} \sum_{m} X_{kapm} \leq 2 \qquad \forall s$$

$$(6) \quad \sum_{(k \in FAC_f \text{ or } FACSTU_f)} \sum_{(a \in GROUP_k)} \sum_{(p \in LUNCHP_k)} \sum_{m} X_{kapm} \leq 2 \qquad \forall f$$

EQUATION EXPLANATION

(1) All courses must be timetabled.

(2) Student group s can attend only one course at a time.

(3) Timetabled courses in category a can not exceed available classrooms.

(4) Faculty f can teach or attend only one class at a time.

(5) Students must have at least one period available for lunch.

(6) Faculty members must have at least one period for lunch.

D. LAGRANGEAN RELAXATION

Consider the following lagrangean relaxation of the original problem which relaxes constraint (2) and doesn't include constraints (5) and (6) or any elastic variables:

MINIMIZE

$$- \sum_{k} \sum_{(a \in GROUP_k)} \sum_{(m \in MODE_k)} \sum_{p} PREF_{kapm} X_{kapm} + \sum_{s} \sum_{p} \sum_{d} \lambda_{spd}$$

$$(1 - \sum_{(k/p \in PER_k \text{ and } k \in COU_s)} \sum_{(a \in GROUP_k)} \sum_{(m \in MODEDAY_d \text{ and } MODE_k)} \sum_{p' \in PERDS_{kp}} X_{kap'm})$$

SUBJECT TO (1), (3) and (4).

Equations (1), (3) and (4) can be solved with a greedy assignment algorithm using the calculated objective function coefficient for X_{kapm}. A greedy algorithm will produce optimal solutions for the relaxed problem only if each faculty member teaches only a single course and classrooms don't become a

limiting resource. This can be seen in the following example. Let's suppose that two courses share the same faculty member, with the following objective related costs for three different periods:

course 1 period 1 : -8 course 2 period 1 : -7

course 1 period 2 : -7 course 2 period 2 : -5

course 1 period 3 : -5 course 2 period 3 : -3.

(for example, assigning course 1 to period 1 would produce a cost of -8).

The assignment that minimizes the sum of the costs and that doesn't produce a faculty (or room) conflict is course 1 period 2 and course 2 period 1 (result -14). The greedy algorithm first timetables course 1 on period 1 and then timetables course 2 on period 2 (result -13) which as shown is not optimal. The fact that a Greedy approach may not produce optimal assignments implies that the Lagrangean relaxation subproblem is not guaranteed to supply a lower bound. Given the difficulty of finding any feasible solution, the calculation of a true lower bound does not seem as useful as the rapid generation of a solution.

The calculation of successive λ_{spd} is done using subgradient optimization, where as suggested by Tripathy (1980)

$$\lambda_{spd}^{r+1} = \max\left[\left(\lambda_{spd}^{r} + \theta_r \gamma_i^r\right), 0\right] \quad \forall spd$$

where

r is the current iteration,

Θ_r is the stepsize for the current iteration. For this problem it was chosen to be

$$\theta_r = 2 \frac{w(\lambda^r)}{\|\gamma^r\|}$$

$w(\lambda^r)$ is the objective function value at the r^{th} iteration,

γ^r is a subgradient vector at the r^{th} iteration. The subgradient used was

$$\gamma_{spd} = 1 - \sum_{(k \,/\, s \in COU_k)} \sum_{(a \in GROUP_k)} \sum_{(m \in MODEDAY_d)} X_{kapm} \quad \forall spd$$

Convergence conditions for this stepsize are (Parker and Rardin, 1988):

$$a)\ \theta^r \geq 0 \ ; \qquad b)\ \lim_{r \to \infty} \theta^r = 0 \qquad \text{and} \qquad c)\ \sum_{r=1}^{\infty} \theta^r = \infty \ .$$

IV. RESULTS

A. DATA SOURCES

The data used to create the test files required for the model correspond to that of the Naval Postgraduate School's Winter Quarter of 1994. The sources to generate the data files are:

From the Registrar:

- Student Groups,
- Offered courses,
- Courses required by students,
- Meeting days.

From the Master Schedule:

- Number of students attending courses,
- Faculty member to teach each course,
- Block structure for courses,
- Required number of hours for a course,
- Classroom where a course may be timetabled.

Other sources:

- The number of classrooms in a category was checked and counted by the author,
- The course preference for periods, modes or rooms are assigned randomly,

- The faculty member requests for periods, days or back to back courses does not correspond to actual requests because the information was lost. It was arbitrarily created by the author to give more reality to the formulation,

- Even though the courses chosen by student groups was obtained from the registrar, the segment assignment was done by the author in a random way. Further research could be done on the analysis of an optimal segment assignment in order to minimize the number of potential conflicts.

It should be noted that the NPS Master Schedule for this quarter doesn't follow the preference constraint of timetabling a course during the same period in 37 cases and that only 19 courses are timetabled in a mode different than those considered in the optimization model. Of those courses comprising the last exception, 12 correspond to three hour courses timetabled using two sessions.

B. SOLVING THE MIXED INTEGER PROGRAM

The Mixed Integer Program presented in Chapter III is generated using GAMS (Gams, 1992) and solved using the X-SYSTEM (Insight, 1990) and XA (Sunset Software Technology, 1994). The option of prereduction was selected for XS.

The dimensions of the problem are:

X-System Prereduction	Before Prereduction	After Prereduction
Discrete Variables	32,235	32,220
Continuous Variables	36,361	212
Total Variables	68,596	32,432
Total equations	43,851	43,574
Nonzero elements	854,106	

None of the mentioned solvers was able to give an optimal solution to the linear programming (LP) relaxation after 48 AMHDAL-5995 CPU hours.

C. CASCADE SOLUTION

Because of the large number of discrete variables involved and the bad result obtained with a straightforward application of standard solvers, a cascade strategy is pursued. This procedure starts by dividing the set of variables involved in the model into some convenient number of subsets. Then the linear programming relaxation of the original problem is solved iteratively adding one subset of variables in every step. At the end of each step all variables that result with a value smaller than 0.05 are fixed to zero and all those variables with value greater than 0.95 are fixed to one.

Finally, the problem containing all variables, including all those that were fixed in a previous stage, is solved.

A convenient partition for the variables is the one that minimizes the number of conflicts between subsets. As a consequence of this kind of partition, the solution of the problem with one subset of variables is as independent as possible from any other solution with a different subset of courses. For the NPS case, it can be considered that the biggest number of conflicts are originated by the student feasibility constraint. With this in mind and because students take most of their courses from their own curriculum, an easy and natural partition of the set of courses is that by Academic Departments. Furthermore, at NPS there are departments whose courses tend to be requested more by students from a different department (conflicts) which means that courses could also be ranked in terms of the number of potential conflicts. This topic is not pursued further by this research but it is estimated that a heuristic based on graph coloring could produce more efficient groupings in terms of their conflicts.

The cascade strategy is used to solve the NPS Winter Quarter timetable explained in the previous section by dividing the total number of courses into nine subsets based on their Academic Department. The models are generated using GAMS (Gams, 1992) and solved using X-SYSTEM (Insight, 1990)

and XA (Sunset Software Technology, 1994). The option of prereduction was selected for XS.

The dimensions of the final mixed integer problem were:

X-System Prereduction	Before Prereduction	After Prereduction
Discrete Variables	29,327	4,445
Continuous Variables	36,361	179
Total Variables	65,688	4,624
Total equations	42,014	19,468
Nonzero elements	1,056,117	
Variables fixed to 1	68	68
Variables fixed to 0	25,006	25,006

None of the mentioned solvers was able to give an optimal solution to the LP relaxation of this final step after using 48 AMHDAL 5995 CPU hours.

No further attempt to solve the mixed integer model is pursued. The results are presented to help motivate a heuristic approach.

D. LAGRANGEAN RELAXATION BASED HEURISTIC

The Lagrangean relaxation based heuristic is programed using IBM AIX XL PASCAL and all computational results obtained using an IBM/RISC/6000 model 590.

Because of poor results when using the stepsize described in Chapter III, the following modifications are used:

- The stepsize is calculated as 100 / $(1.11)^r$, where r is the iteration number,
- After running 500 iterations, the solution with the least number of student conflicts is chosen. From this best solution, the objective function coefficients ($PREF_{kapm}$) corresponding to the best combinations (course-classroom-period-mode) are changed to make it more desirable to the greedy algorithm,
- After six consecutive 500 iteration runs, a final basic timetable heuristic is applied to the conflicting courses. This heuristic is based on the best solution and consists of first finding for every course the combination of period and day that produces the smallest number of days in conflict. This combination is used to find periods and days where to timetable the remaining number of hours to complete the number required. The steps of choosing the best combination and completing the number of required hours are done considering the corresponding faculty members availability. The order to revisit courses is that of the best solution. A final step of the heuristic finds an available classroom for the course on the days and periods previously determined.

The final algorithm is:

```
stepsize ← 100
for j : 1 to 6
     for i : 1 to 500
          Use a greedy approach to timetable courses,

          Calculate number of student violations produced with this
          assignment,

          Calculate new λ,
          Stepsize ←  Stepsize / 1.11,

          If violations < Best
          Best ← violation
               OptAssign ←  Assignment.

     Reduce cost of non-conflicting combinations from OptAssign,

     Reset stepsize to 5 and λ to 0.

Use simple assignment heuristic to solve conflicting combinations of
the Best solution and assign available classrooms to all courses.
```

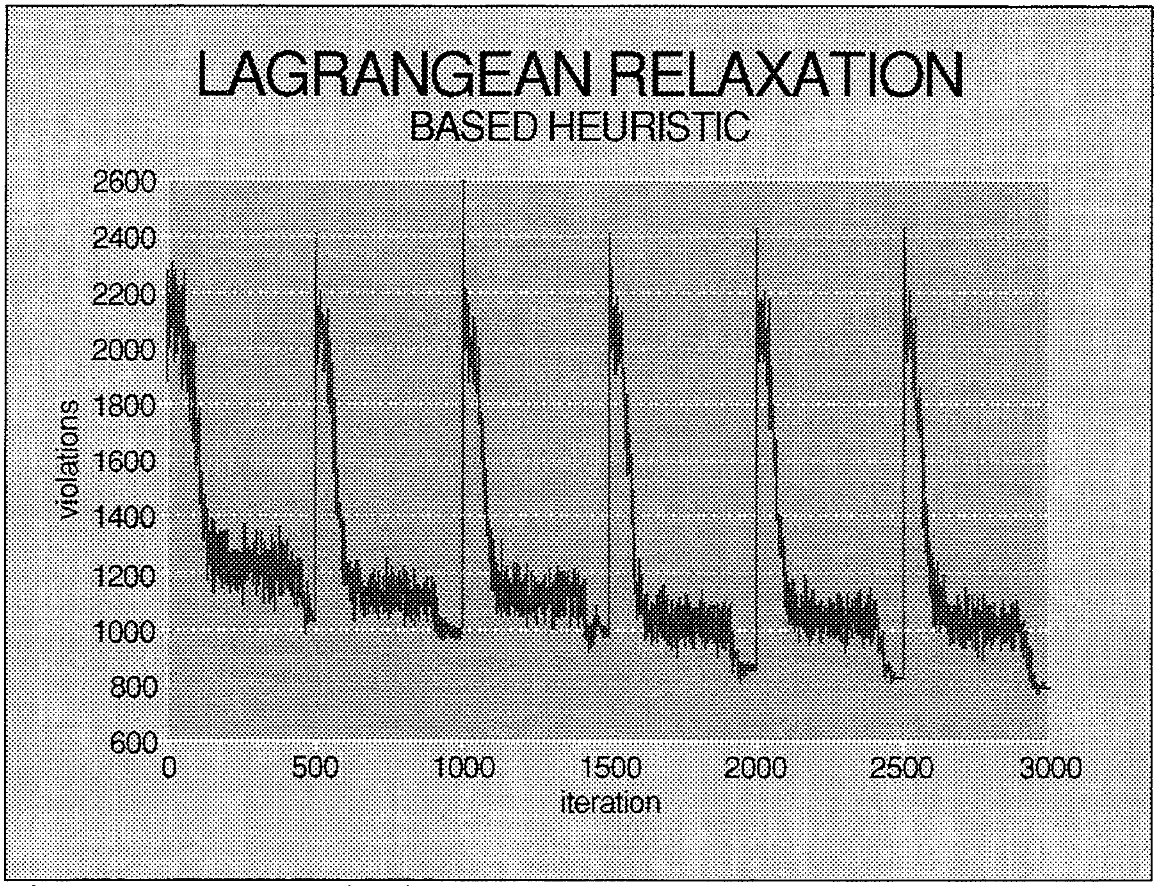

Figure 1 The heuristic's reduction in student conflicts by iteration. The objective function contains the relaxed constraint measuring student conflicts which when satisfied reduces violations to zero.

The heuristic's progress can be observed in terms of the number of student conflicts on Figure 1 and in terms of the number of conflicting courses on Figure 2.

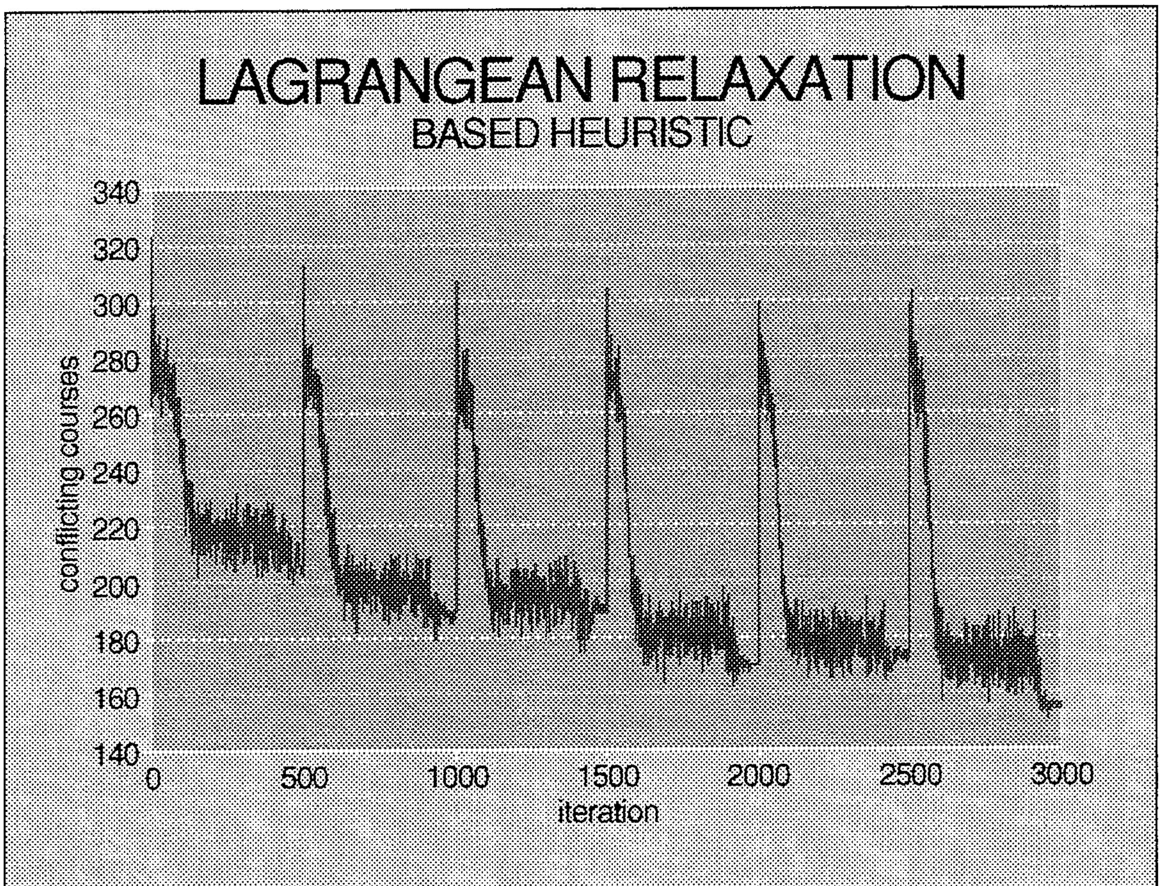

Figure 2 The heuristic's reduction in course conflicts by iteration. The objective function contains a relaxed constraint which when satisfied will reduce conflicting courses to zero.

The measures of effectiveness for the algorithm performance are:

- CPU time per iteration and total CPU time,
- total number of iterations,
- number of courses not timetabled due to student conflicts,
- number of courses timetabled using modes as defined in Chapter III,
- number of courses without a room assignment due to non-availability of classrooms for the periods when a course can be timetabled without student conflicts. Also

presented is the number of hours for courses without a room assignment.

The final results of the algorithm are summarized below:

- Problem generation CPU time : 160 min.
- CPU time per iteration : 12.6 sec.
- Total CPU time : 790 min.
- Lagrangean Relaxation iterations : 3000 iter.
- Total Number of courses : 470 courses.
- Courses not timetabled : 28 courses.
- Courses timetabled using modes : 381 courses.
- Courses without a room assignment : 2 courses.
- Course-hours without a room : 8 hours.

E. POLICY STUDIES

The following policy studies were conducted using the Lagrangean relaxation based heuristic explained in the previous section and the NPS 1994 Winter Quarter data.

1. Classroom Reduction

Two variations were investigated. The first one eliminates Root Hall rooms availability (total of 16 rooms). The results were:

- Courses not timetabled : 28 courses,
- Course-hours without a room : 75 hours.

The second test consisted in making Glasgow Hall first floor unavailable for teaching (total of 10 rooms). The results were:

- Courses not timetabled : 28 courses,
- Course-hours without a room : 172 hours.

2. Student Population Variations

The effects on the timetabling problem of an increment in the student population was roughly simulated by randomly eliminating 10% of the School's lecture classrooms (total of seven rooms). The results of the algorithm were

- Courses not timetabled : 28 courses.
- Course-hours without a room : 18 hours.

The effects on the timetabling problem of a reduction in the student population was simulated by randomly decreasing the number of student groups by 10% (95 student groups, 3240 equations). The results of the algorithm were:

- Courses not timetabled : 21 courses.
- Courses timetabled using modes : 390 courses.
- Courses without a room assignment : 4 courses.
- Course-hours without a room : 18 hours.

3. Analysis

The results show an increased number of courses without a room assignment and a constant number of courses not timetabled whenever the number of classrooms is reduced and no changes are made to the student population. The increased number of courses not timetabled reflects the difficulty of the timetabling problem when classroom reductions are made and it can be used as a comparative measure when making this kind of study. It also indicates the need to augment the greedy heuristic to check for classroom availability when it searches for the most preferred course (this check was found initially to be unnecessary).

Reducing student groups did not significantly decrease the difficulty of the NPS timetabling problem since it appears that student conflicts do not decrease significantly. This was apparently due to the random selection of student groups to be eliminated.

V. CONCLUSIONS

The NPS quarterly course timetabling problem is difficult. This is apparent when observing the number of integer variables and equations when the MIP is formulated for the 1994 Winter Quarter (even after a significant reduction was obtained by using grouping techniques there were still 32,235 discrete variables and 43,851 equations). This size is greater than any other mentioned in the researched literature.

The problem was solved for the aforementioned period using a Lagrangean relaxation based heuristic for all but 28 of the 470 offered courses, with only two courses having no room assignment. The heuristic was used to solve two policy studies concerning student population and classroom availability showing the increasing difficulty of the timetabling problem for these situations. These studies also suggest future research is needed to modify the heuristic if classrooms become a limiting resource.

APPENDIX

Following are all NPS classroom categories with their common characteristics divided by buildings.

BUILDING	CATEGORIES	ROOMS	TOTAL	CHARACTERISTICS
BULLARD HALL	A1	B104 B202	2	Capacity < 21
	A2	B100A	1	Lab
	A3	B201	1	Lab
	A4	B208	1	Lab
	A5	BB14	1	Lab

BUILDING	CATEGORIES	ROOMS	TOTAL	CHARACTERISTICS
GLASGOW HALL	A6	G015	3	Capacity < 28
		G017		
		G117		
	A7	G014	11	Capacity < 36
		G018		
		G019		
		G110		
		G113		
		G114		
		G115		
		G118		
		G129		
		G130		
		G133		
	A8	G122	1	Capacity < 44
	A9	G109	1	Capacity < 180
	A10	G303	6	NSA conference
		G306		
		G386		
		G387		
		G388		
		G389		
	A11	G128	1	Lab
	A12	G203	1	Lab
	A54	G302	1	Lab

BUILDING	CATEGORIES	ROOMS	TOTAL	CHARACTERISTICS
HALLIGAN HALL	A13	H109	7	Capacity < 33
		H121A		
		H121B		
		H123		
		H125		
		H201E		
		H201F		
	A14	H138	1	Lab

BUILDING	CATEGORIES	ROOMS	TOTAL	CHARACTERISTICS
INGERSOLL HALL	A15	I263	4	Capacity < 28
		I285		
		I381		
		I386		
	A16	I323	5	Capacity < 32
		I325		
		I366		
		I369		
		I387		
	A17	I119	4	Capacity < 36
		I265		
		I280		
		I282		
	A18	I267	2	Capacity < 40
		I322		
	A19	I271	1	Capacity < 46
	A20	I260	1	Capacity < 51
	A21	I158	1	Lab
	A22	I250		Lab

BUILDING	CATEGORIES	ROOMS	TOTAL	CHARACTERISTICS
ROOT HALL	A23	M112	1	Capacity < 24
	A24	R228	1	Capacity < 20
	A25	R208	5	Capacity < 30
		R240	1	
		R242	1	
		R256	1	
		R260	1	
	A26	R109	1	Capacity < 35
	A27	R113	1	Lab
	A28	R123	1	Lab
	A29	R125A	1	Lab
	A30	R262	1	Lab

BUILDING	CATEGORIES	ROOMS	TOTAL	CHARACTERISTICS
SPANAGEL HALL	A31	S138 S140 S226 S248 S310 S342	6	Capacity < 21
	A32	S332 S408 S429	3	Capacity < 28
	A33	S136	1	Capacity < 37
	A34	S208 S221 S316	3	Capacity < 51
	A35	S117 S231 S321	3	Capacity < 63
	A36	S421	1	Capacity < 78
	A37	S006	1	Lab
	A38	S019	1	Lab
	A39	S025	1	Lab
	A40	S107	1	Lab
	A41	S111	1	Lab
	A42	S125	1	Lab
	A43	S127	1	Lab
	A44	S135	1	Lab
	A45	S263	1	Lab
	A46	S301	1	Lab
	A47	S303	1	Lab
	A48	S341	1	Lab
	A48	S419	1	Lab
	A50	S431	1	Lab
	A51	S543	1	Lab
	A52	S612	1	Lab
	A53	S703	1	Lab

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